

Name _____ Test 3, Spring 2021

1) Find all the eigenspaces of the matrix below (15 points)

$$\begin{bmatrix} -1 & -3 & 3 \\ 0 & -1 & 0 \\ 0 & -3 & 2 \end{bmatrix}$$

2) Find the diagonalization of the matrix from the previous problem. (5 points)

(If you couldn't solve the previous problem, make up an answer to answer this problem)

3) Given the basis below, find an orthogonal basis for the same vector space. (10 points)

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\}$$

4) Answer the following questions. (3 points each)

A) Let A be a 5×5 with eigenvalues $0, 0, 1, 2, 3$. What is the maximum rank of A ?

B) Let A be a 3×5 matrix whose nullity is 4. When row reduced, how many rows of zeroes are there?

C) Consider a system of 4 equations and 4 variables that has a unique solution. When row reduced, how many pivots does the corresponding matrix have?

D) Let A be a 6×6 matrix whose corresponding linear transformation T is onto. Is T one-to-one?

E) Let A be a 3×3 matrix whose corresponding linear transformation T is not one-to-one. What is the determinant of A ?

5) Given the basis and vector below, find a formula for the vector \vec{x} in standard coordinates. (10 points)

$$B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \end{bmatrix} \right\}; [\vec{x}]_B = \begin{bmatrix} 5 \\ 3 \end{bmatrix}_B$$

6) Find a formula for for $[T]$ given the facts below. (5 points)

$$T \left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 5 \\ 6 \end{bmatrix}; T \left(\begin{bmatrix} 3 \\ 4 \end{bmatrix} \right) = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

7) Find the null space of the matrix below. (10 points)

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

8) Reduce the matrix below to reduced echelon form. (10 points)

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 6 \\ 0 & 1 & 3 & 4 & 1 \end{bmatrix}$$

9) Let A be a 5×5 matrix with eigenvalues $0, 1, 2, 3, 4$. Is A diagonalizable? (5 points)

10) Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ and $f(x) = x^2 + 5$. Find $f(A)$. (5 points)

11) Find the product below. (5 points)

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 & 2 & 2 \\ 3 & 1 & 3 & 3 & 3 \\ 4 & 4 & 1 & 4 & 4 \\ 5 & 5 & 5 & 1 & 5 \\ 6 & 6 & 6 & 6 & 1 \end{bmatrix}$$

12) Find the inverse of the matrix below. (5 points)

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 4 \\ 0 & 1 & 0 \end{bmatrix}$$